DIGITAL I/O DRIVEN SWITCH TIMING ANALYSIS

Purpose: This application note presents the timing of the process of proceeding through a digital I/O driven switch in a QuickStick system.

Introduction

The process of moving through a switch can cause delays in a system, which can in turn lead to throughput reductions. As a result, optimizing switching is a major concern in systems where optimized throughput is a goal. This document will present the process of switching and the relationship of the parameters involved. Using the equations presented herein, it should be possible to optimize the various parameters involved to minimize the time required to transit a switch.

Configurable Parameters

When initially setting up the switch, there are a series of parameters that must be set in the configuration file that will affect the transit time of the switch. The configuration parameters that will affect throughput are:

1. Keep out areas (k) – Areas that the vehicle will not enter unless it can proceed all the way though them. These areas are generally placed before a switch to prevent collisions with the switch mechanism or other vehicles that are transiting the switch when the vehicle arrives. This area will define where a vehicle stops to wait and where the vehicle will request entry into the node. *For the purposes of the calculations below, we will use the length of the keepout area, rather than the point it is configured from.*
2. Clearance distances (d_c) – The distance from the end of the motor bordering the node that the vehicle needs to be to be clear of the mechanism and any potential traffic crossing behind it. This applies only when leaving the switch.
3. Downstream Gaps (g) - This is the distance between the end on one motor and the beginning of the next. This affects how far the vehicle needs to travel to cross the switch. In the case of a diverge node, where the gap downstream of the last motor on the previous path may be different, a *gap delta* (g_d) is added to accommodate this difference.

More information about how these parameters relate to switch configuration and the logic guiding how these parameters are determined is defined in separate documentation. Each of the parameters are shown in the figures below:

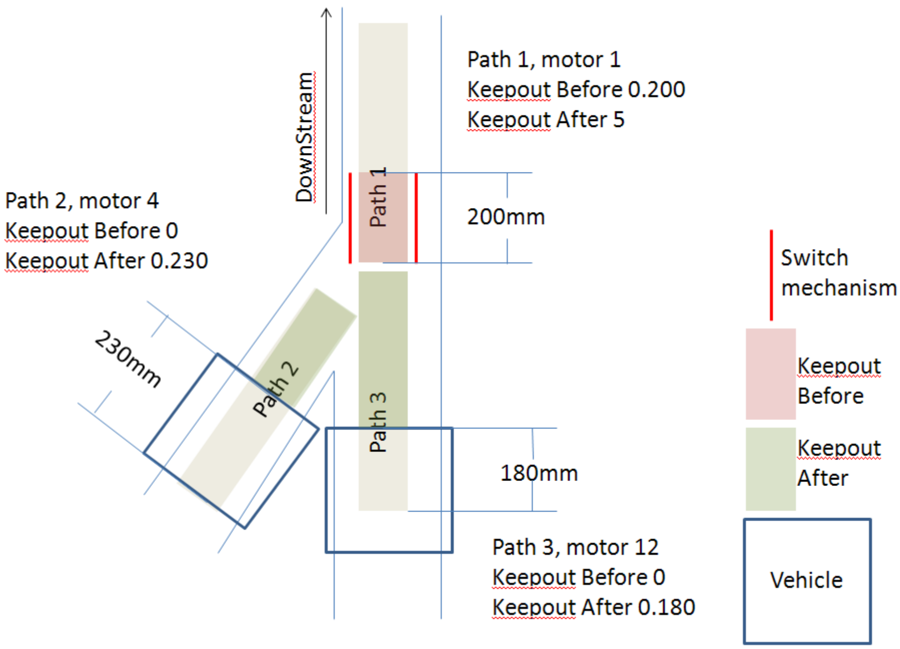

Figure 1: Merge Keep Out Areas

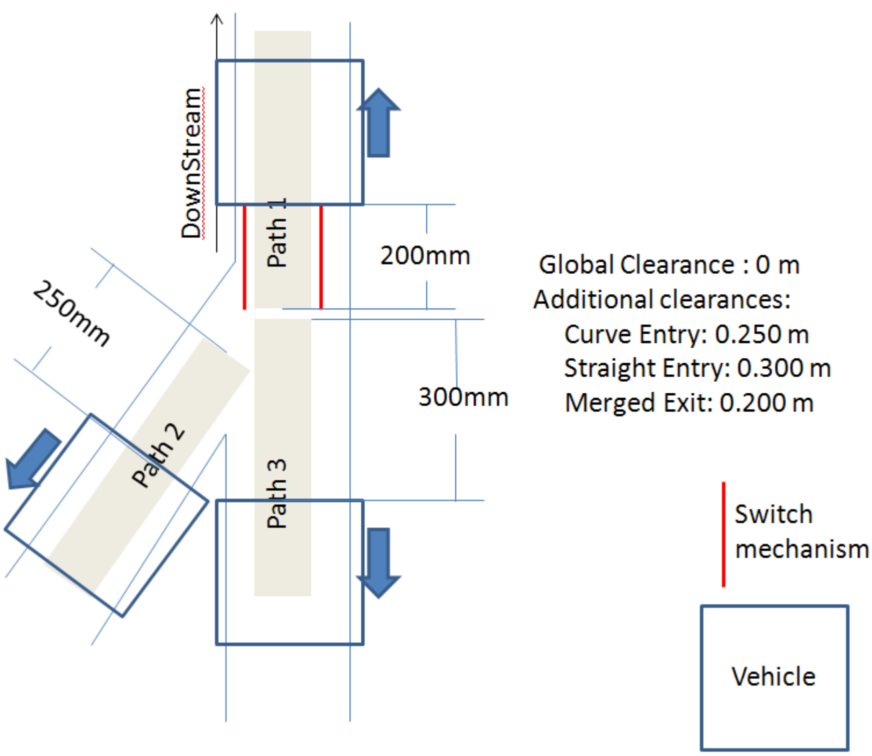

Figure 2: Merge Clearance Distances

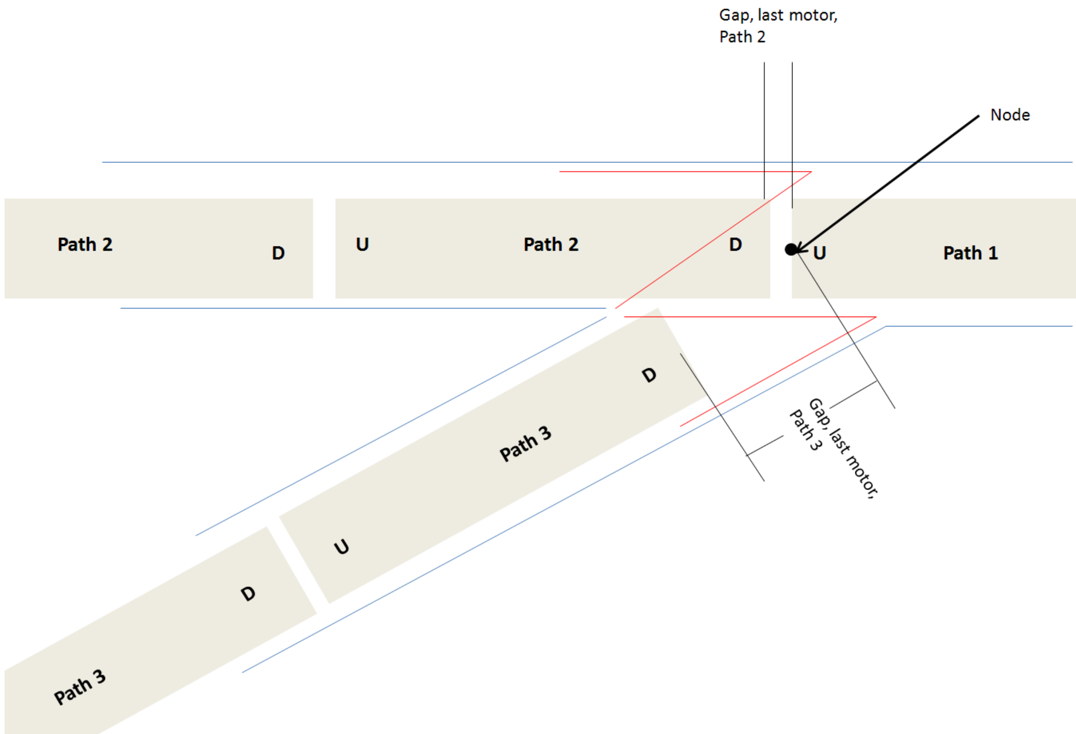

Figure 3: Merge Downstream Gaps

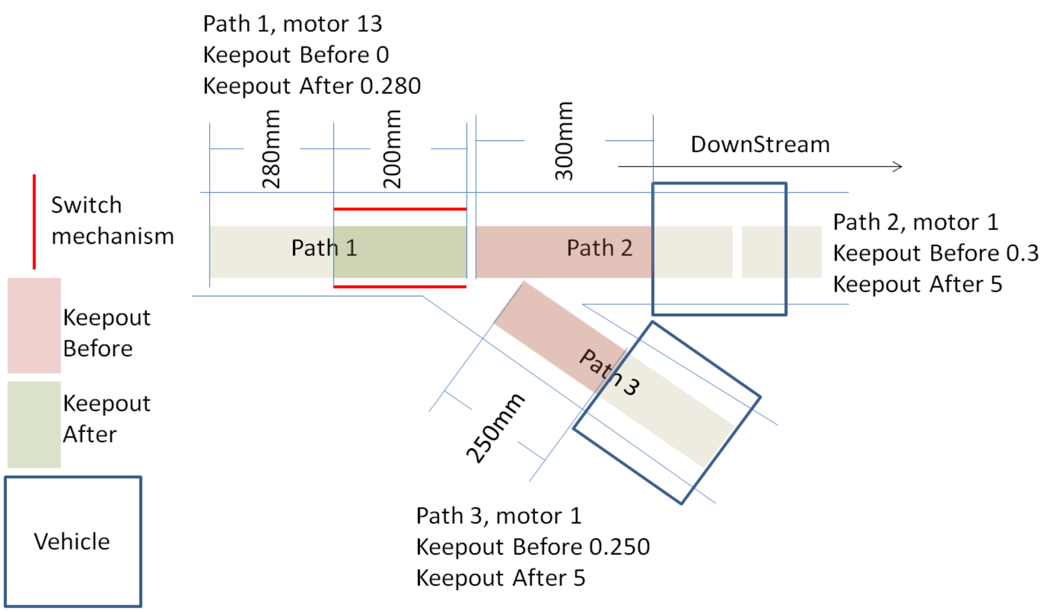

Figure 4: Diverge Keep Out Areas

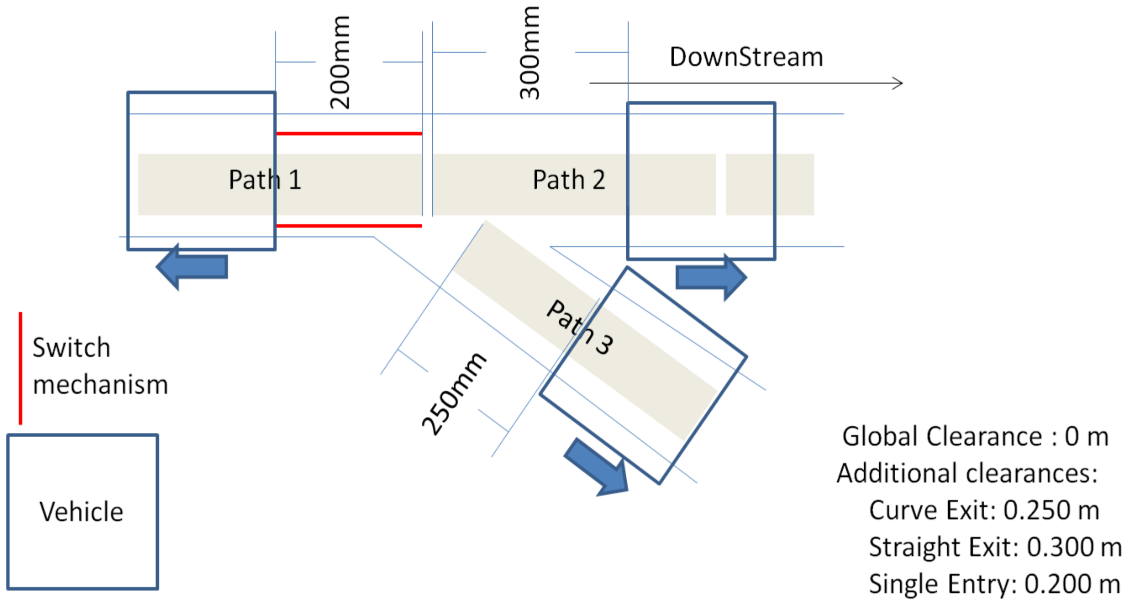

Figure 5: Diverge Clearance Distances

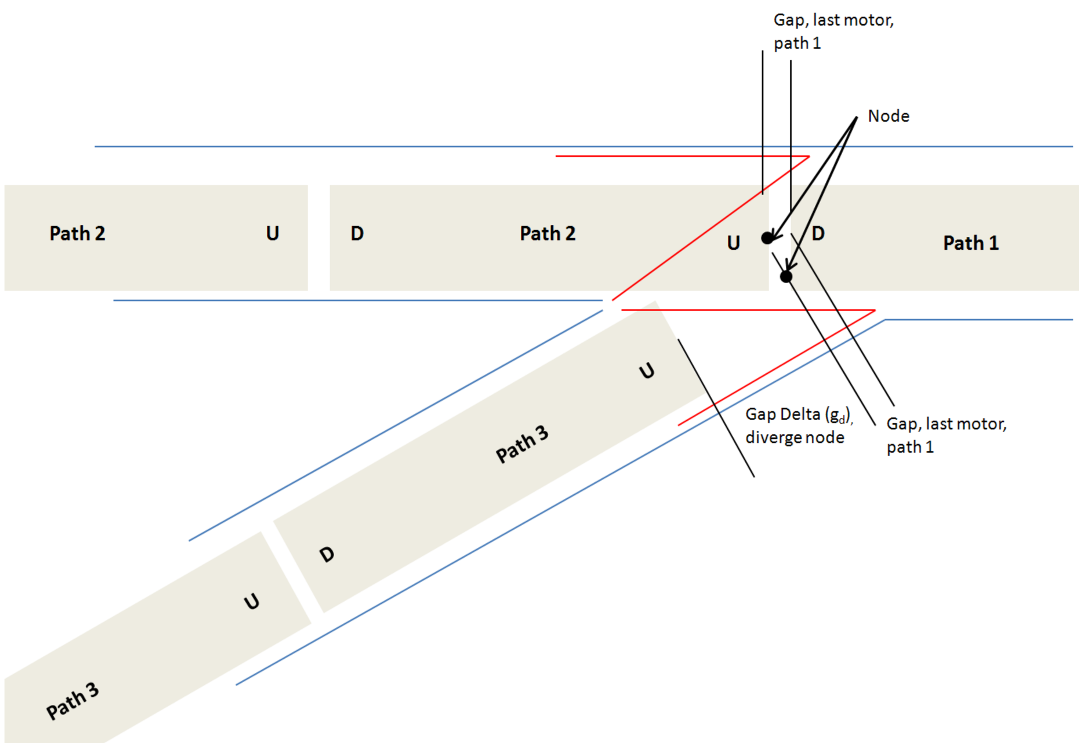

Figure 6: Diverge Downstream Gaps

Node Transit Timing

The process of moving through a node can be broken down into seven distinct sections, with seven defined points at which an event occurs that furthers the process. These critical points are as follows denoted by T (for time) followed by a subscript representing the order in which they occur.

T_0 – Permission to enter the node is requested
T_1 – The Node Controller Digital Output to the switch which corresponds to the direction that needs to be taken is energized
T_2 – The switch energizes the Node Controller Digital Input corresponding to the direction to be taken. The Node controller checks that the digital input corresponding with the direction to be taken is energized AND the Node controller checks the digital input corresponding with the alternate direction to ensure that it is de-energized.
T_3 – The vehicle is granted permission to enter the node
T_4 – The trailing end of the vehicle passes the clearance distance from the node
T_5 – The Node Controller Digital Output to the switch which corresponds to the direction that needs to be taken is de-energized. The Node is clear and another vehicle can begin the entry process.
T_6 – The switch de-energizes the Node Controller Digital Input corresponding to the direction that was taken.

These steps are shown in Figure 7 with the state of the Digital IO on the node controller at that step.

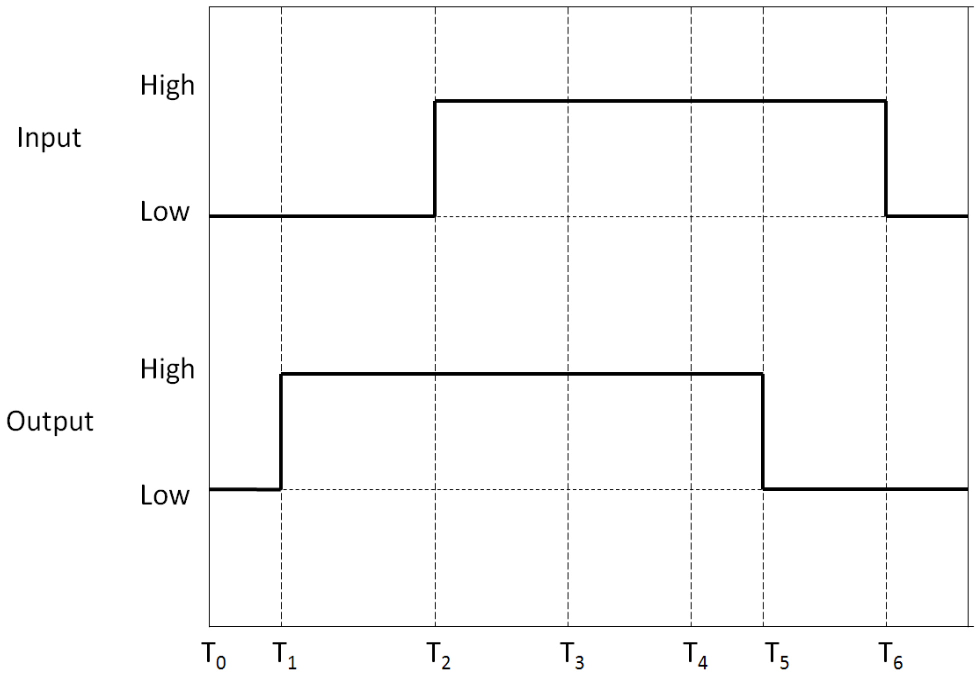

Figure 7: Timing Diagram of the Switching Process

Explanation of Conventions in Equations

Distance-All distances are in meters, with negative distances before the switch and positive distances after. The zero position is equal to the position of the node.

Time- All times are in seconds.

P_0 – The distance from the center of the vehicle to the node at time zero, in meters.

V_0 – The velocity of the vehicle at time zero in m/s

a_c – the commanded acceleration.

a_a – the maximum achievable acceleration. This is dictated by the available thrust, mass of the vehicle, and drag coefficient. If higher than the commanded acceleration, use the commanded acceleration for this value.

L_b – the length of one motor block. This is 96mm for QS100.

L_v – length of the vehicle

V_c - commanded velocity during the movement

All accelerations will be considered to be infinite jerk (constant acceleration) for ease of computation.

All equations assume that the vehicle will not need to decelerate for collision avoidance before leaving the switch.

Permission Distance (*p*)

Headway Distance is the required separation of vehicles while moving to allow them to stop at maximum defined acceleration without colliding. This distance will be maintained in front of any moving vehicle in order to ensure collision avoidance. It is equal to the stopping distance plus the longest block between the vehicles, which will be the one at the end of the motor that includes the gap length (g). The block length is added to because the preceding vehicle must clear a block before the following vehicle can enter. It can be found mathematically as:

$$h = \frac{v^2}{2a_c} + L_b + g$$

The blocks within this distance are allocated to the moving vehicle that is generating the headway area. Requests for permission in this space are requested at the permission distance (p) in front of the vehicle, which is the minimum stopping distance plus the distance the vehicle will travel in 30ms. Due to communications and processing time, the request is generally processed 10ms before the vehicle reaches the minimum stopping distance from the end of the last block it acquired permission to enter. This 10ms window will be referred to as the headway area lead time (T_{hl}). This permission distance is represented mathematically as follows:

$$p = \frac{v^2}{2a_c} + vT_{hl}$$

T_0-Initial Position and Velocity

At T = 0 (T_0), the vehicle is positioned such that it is requesting permission to cross the node. If a keep out area is in place, this will be the position where the vehicle requests to enter the keep out area. This position is dictated by the size of the headway area in front of the vehicle. The distance will the equal to the length of the keepout area (k) added to the distance from the keepout zone that the vehicle requests permission to enter it, which is the permission distance (p). If approaching from upstream, the gap length (g) will also be added. If approaching from downstream direction on a curved path into a diverge node, the downstream gap delta (g_d) must be added. This position can be found mathematically as:

Approaching from:
Upstream: $P_0 = -k - p - g - \frac{L_v}{2}$
Downstream: $P_0 = -k - p - \frac{L_v}{2}$
Downstream into Diverge from curve: $P_0 = -k - p - g_d - \frac{L_v}{2}$

The velocity of the vehicle at this point will be denoted as V_0. This velocity can vary based on traffic and stop locations. In most cases, this velocity will be the commanded velocity (V_c). If a station is located close to the switch, the vehicle may not have sufficient distance to accelerate to the commanded velocity and V_0 will be less than the commanded velocity as a result.

Period T_0 to T_1 (T_{01})

In the time between T_0 and T_1, the node controller will process the request for the node and set the digital output corresponding to the direction that the vehicle has been commanded to take high. In this calculation, processing time can be considered negligible. The time for the Digital Input/Output (DIO) board to process the input from the motherboard and set the output high is approximately 0.1 ms (T_{dio}). During this time, the vehicle will remain traveling at full speed, if the headway area lead time (T_{hl}) discussed previously has not expired.

$$T_{01} = T_1 = T_{dio}$$

If $T_1 < T_{hl}$

$$P_1 = P_0 + V_0 T_{01}$$

$$V_1 = V_0$$

If $T_1 > T_{hl}$

$$P_1 = P_0 + V_0 T_{01} - \frac{1}{2} a_c (T_1 - T_{hl})^2$$

$$V_1 = V_0 - a_c (T_1 - T_{hl})$$

Period T_1 to T_2 (T_{12})

In the time between T_1 and T_2, The Host-designed switch will need to register the output of the node controller being set high, mechanically move the switching mechanism, and set the input line to the node controller high once the mechanism is in place. The key point for the progression of the vehicle through the switch is the changes to the input line, which will drive the process forward whether the switch is mechanically ready or not. The time between the output being set high and the input being set high is known as $T_{\text{switch up}}$ (T_{su}).

$$T_2 = T_1 + T_{su}$$

$if\ T_2 < T_{hl}$:

$$P_2 = P_1 + V_0(T_{su})$$

$$V_2 = V_1$$

$if\ T_2 > T_{hl}$:

$$P_2 = P_1 + V_1(T_{su}) - \frac{1}{2}(a_c)(T_2 - T_{hl})^2$$

$$V_2 = V_1 - (a_c)(T_2 - T_{hl})$$

Period T_2 to T_3 (T_{23})

In the time between T_2 and T_3, the node controller reacts to the input being set high and grants the vehicle permission to move to the other side of the node. This period begins with the DIO board processing the high signal on the input line and passing that information to the node controller, which takes the same T_{dio} defined previously. The motherboard polls the input from the DIO board every 40ms, so in the worst case it would take 40 ms (T_{poll}) for the high input from the DIO board to be registered in the motherboard and reacted to. This is then followed by, in the worst case, 1ms of processing time and 5ms of time communicating the granted permissions out to the motor. This 6ms time period will be referred to as $T_{proc.}$ Additionally, if the input from the opposite direction (the one that is not being taken) has not been set low, the node controller will wait until it has to grant permission to cross the node. This time spent waiting will be referred to as T_{wait}.

$$T_{23} = T_{dio} + T_{poll} + T_{proc} + T_{wait}$$

$$T_3 = T_2 + T_{23}$$

$if\ T_2 > T_{hl}$:

$$P_3 = P_2 + V_2(T_{23}) - \frac{1}{2}(a_c)(T_{23})^2$$

$$V_3 = V_2 - (a_c)(T_{23})$$

$if\ T_2 < T_{hl}$:

$$P_3 = P_2 + V_2(T_{23}) - \frac{1}{2}(a_c)(T_{23} - (T_{hl} - T_2))^2$$

$$V_3 = V_2 - (a_c)(T_{23} - (T_{hl} - T_2))$$

Period T_3 to T_4 (T_{34})

In the time between T_3 and T_4, the vehicle moves through the switch until the rear of the vehicle passes the defined clearance distance (d_c). In this case the initial velocity (V_3) and position (P_3) are known, as is the final position (P_4). If the vehicle is below its commanded velocity (V_c), it will accelerate back up to it while moving through the switch. This creates three possible cases, based on what acceleration period is required to reach the commanded velocity:

$$P_4 = d_c + \frac{L_v}{2}$$

$P_4 = d_c + \frac{L_v}{2} + g_d$ only if exiting a diverge node into the curve

$$d_{34} = P_4 - P_3$$

$$V_c^{\,2} = V_3^{\,2} + 2a_a d_{acc}$$

$$d_{acc} = \frac{V_C^{\,2} - V_3^{\,2}}{2a_a}$$

If $d_{acc} = 0$:

$$V_4 = V_3 = V_c$$

$$d_{34} = V_3 T_{34}$$

$$T_{34} = \frac{d_{34}}{V_3}$$

$$T_4 = T_3 + T_{34}$$

If $0 < d_{acc} < d_{34}$:

$$V_c = V_3 + a_a T_{acc}$$

$$T_{acc} = \frac{V_c - V_3}{a_a}$$

$$V_4 = V_3 + a_a T_{acc} = V_c$$

$$d_{34} - d_{acc} = V_c (T_{34} - T_{acc})$$

$$T_{34} = \frac{d_{34} - d_{acc}}{V_4} + T_{acc}$$

$$T_4 = T_3 + T_{34}$$

If $d_{acc} > d_{34}$:

$${V_4}^2 = {V_3}^2 + 2a_a d_{34}$$

$$V_4 = \sqrt{{V_3}^2 + 2a_a d_{34}}$$

$$V_4 = V_3 + a_a T_{34}$$

$$T_{34} = \frac{V_4 - V_3}{a_a}$$

Period T_4 to T_5 (T_{45})

In the time between T_4 and T_5, the node controller notes that the vehicle has cleared the node and sets the output corresponding to the direction that was taken low. This action also clears the node and allows for the entry of another vehicle. The Node controller receives vehicle status messages every 2ms, so considering worst case timing, it will take 2ms for the node controller to note that the vehicle has cleared the node (T_{status}). After a negligible processing time, it will then take T_{dio} for the output board to process the signal from the motherboard and set the output low. There are three possible equation sets that can be used here based on whether or not the vehicle has finished its acceleration in the previous time period. Regardless of the acceleration requirements the time to complete this section of the process will be constant.

$$T_{45} = T_{status} + T_{dio}$$

$$T_5 = T_4 + T_{45}$$

$$V_c = V_4 + a_a T_{acc}$$

$$T_{acc} = \frac{V_c - V_4}{a_a}$$

If $T_{acc} < T_{45}$:

$$V_5 = V_4 + T_{acc} a_a = V_c$$

$$P_5 = P_4 + V_4 T_{acc} + \frac{1}{2} a_a T_{acc}^{\ 2} + V_c (T_{45} - T_{acc})$$

If $T_{45} < T_{acc}$:

$$V_5 = V_4 + T_{45} a_a$$

$$P_5 = P_4 + V_4 T_{45} + \frac{1}{2} a_a T_{45}^{\ 2}$$

Period T_5 to T_6 (T_{56})

In the time between T_5 and T_6, the system returns to its previous state with the input to the node controller that was raised lowered. This is no longer related to the vehicle that passed previously, but needs to be completed before any following vehicles can go in the other direction from the one followed during this cycle. If this step is not completed, the next vehicle will incur waiting time at T_3. The time between the output being set low and the input being set low is known as $T_{\text{switch down}}$ (T_{sd}). This may not be the same time as the switch being completely physically lowered, depending on the design of the switch. This section also contains the processing time for the DIO board to report the input being set low to the motherboard (T_{dio}).

$$T_{56} = T_{sd} + T_{dio}$$

$$T_6 = T_5 + T_{56}$$

$$V_c = V_5 + a_a T_{acc}$$

$$T_{acc} = \frac{V_c - V_5}{a_a}$$

If $T_{acc} < T_{56}$:

$$V_6 = V_5 + T_{acc} a_a = V_c$$

$$P_6 = P_5 + V_5 T_{acc} + \frac{1}{2} a_a {T_{acc}}^2 + V_6 (T_{56} - T_{acc})$$

If $T_{56} < T_{acc}$:

$$V_6 = V_5 + T_{56} a_a$$

$$P_6 = P_5 + V_5 T_{56} + \frac{1}{2} a_a {T_{56}}^2$$

Effect on Following Vehicles

A following vehicle's ability to begin this process will depend on the status of the vehicle in front of it. If the vehicle in front has released the node (passed T_5) when the following vehicle reaches the point where it initially requests the node (the first vehicle's P_0, referred to in this section as P_r or point of request), there will be no effect on the passage of the following vehicle and it will proceed like the first vehicle.

If the first vehicle has not released the node, then the following vehicle will begin decelerating after the headway area lead time has elapsed. This will have the following effect on the P_0 and V_0 of the following vehicle. This effect will depend on the time difference between when the following vehicle reaches the point of request and when the first vehicle releases the node, referred to here as T_{clear}.

$$P_0 = P_r + V_r(T_{clear}) - \frac{1}{2} a_c (T_{clear} - T_{hl})^2$$
$$V_0 = V_r - a_c(T_{clear} - T_{hl})$$

With the lowest possible values being the following depending on what direction the vehicle is approaching from.

Approaching from:
Upstream: $P_0 = -k - g - \frac{L_v}{2}$
Downstream: $P_0 = -k - \frac{L_v}{2}$
Downstream into Diverge from curve: $P_0 = -k - g_d - \frac{L_v}{2}$

$$V_0 = 0$$

A corner case of this is when the node is released after the vehicle passes the point of request, but before it reaches the headway distance and begins to slow. In this case, the original equations can be used, but with the normal T_{hl} value replaced with that value minus the time elapsed since passing the point of request.

Summary

These equations should provide an estimate as to the switching time and the process involved. T_5 can be considered to total time unless the following vehicle arrives early enough that it requires T_6 to occur before it can enter. In either case, $P_5 – P_0$ can be considered the total distance key to switching.

This representation will not be wholly accurate due to the assumption of constant acceleration and the taking of the maximum polling time as the exact detection time. The inexact nature of the value used for the actual acceleration will also affect the output of these equations. All results should be validated on the system in question.

Please contact MagneMotion technical support with any questions on the material presented above.

Appendix 1: Programmer's Key information

1) When does the switch get tripped?
   When the vehicle reaches the permission distance from the edge of the keep out zone
2) What does the vehicle do if the switch is occupied or is not in position?
   The vehicle plans to stop with nose just outside of the keepout area and begins to decelerate accordingly.
3) Once the switch is tripped, how long does it take to get into position?
   This is determined primarily by the host hardware. For the total time, add the time for the 6ms update cycle, 1ms processing time, DIO output switching time, host switching time, DIO input processing time, and up to 40ms for polling of DIO input. There can then be up to 6ms of delay for next request to come in and be granted.
4) Where does the vehicle clear the switch?
   The switch will clear once the rear edge of the vehicle passes the clearance distance.

More Information
MagneMotion website: www.magnemotion.com
Questions & Comments: http://www.magnemotion.com/about-magnemotion/contact.cfm